Brief discussion of arrow penetration

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Poncelet description

The simple Poncelet Equation works well for penetration of many materials, especially for soils, and especially less-than-fully dense sand and clay. Force on the projectile is

$$F = M\frac{dV}{dt} = -(\rho C V^2 + R)A$$

Thus projectile slows as:

$$V = \left[\left(\frac{R}{\rho C} + V_0^2\right)e^{-\frac{2\rho C A x}{M}} - R/\rho C\right]^{1/2}$$

And the total penetration can be computed from

$$P = \frac{M}{2\rho C A}\left[\ln\left(\frac{\rho C V_0^2}{R}\right) + 1\right]$$

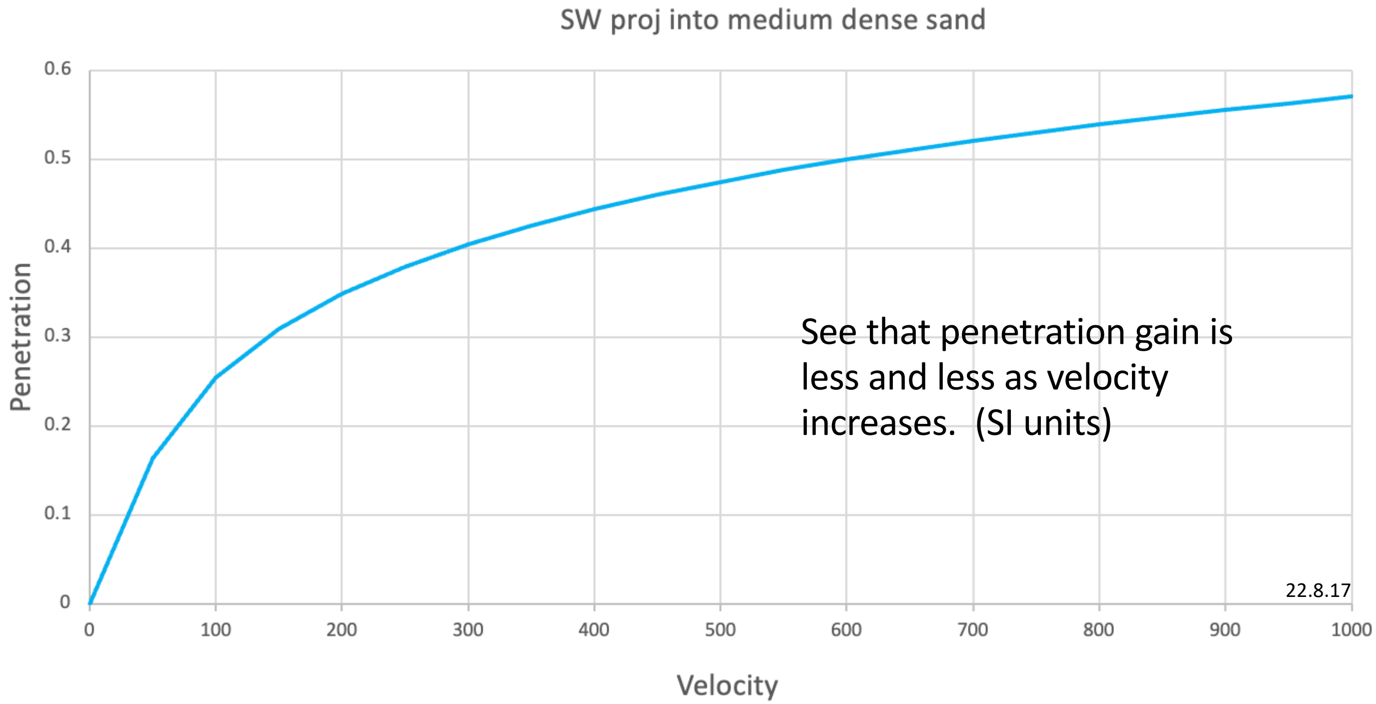
Here R is the target strength, C is the drag coefficient, M is projecitile mass, A is projectile cross section area, rho is target density. Subscript 0 is the impact velocity, and x is penetrated distance.

Poncelet shows penetration becomes less efficient at high velocity: P(V) example of sand

Using data for medium dense sand and description of recent projectiles tested at SwRI::

M=0.104 kg　　　　　　　　Density = 1740 kg/m^3

D=.025m　　　　　C=0.9



Penetrator Efficiency

What is best penetrator?
Consider constraint of constant kinetic energy (KE)
This is often the case for guns. Probably for bows too.
For a given shape - what is the best velocity
For a given velocity, what's the best shape
shape parameter is fineness ratio f=L/D

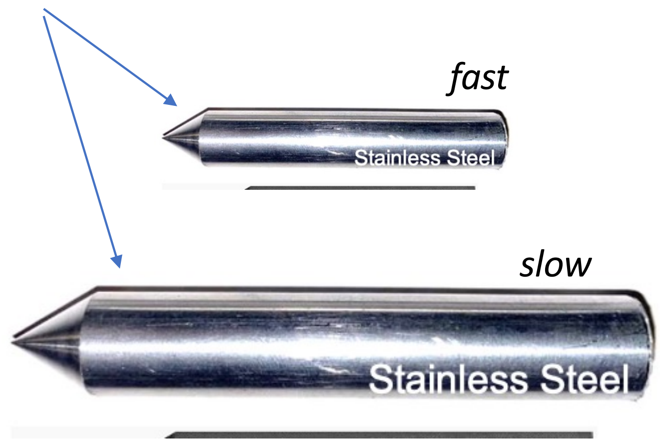


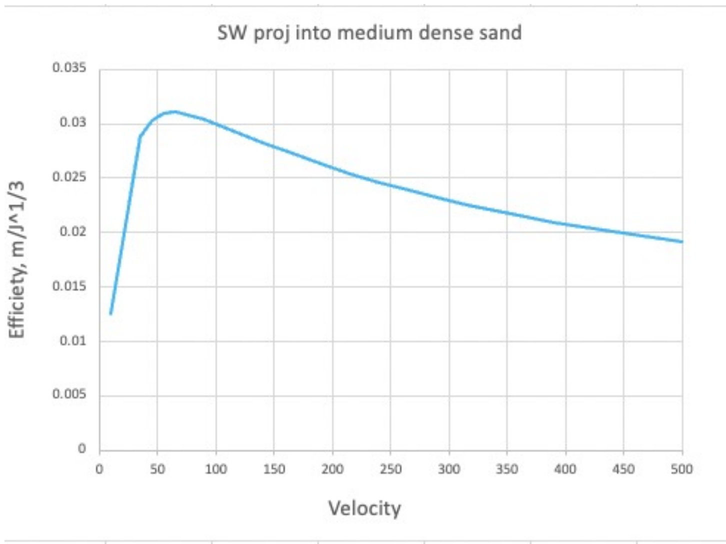
Figure of merit is penetration/cu root KE

$$\frac{P}{E^{1/3}}=\frac{(\rho f)^{2/3}}{\pi^{1/3}\rho_s C V^{2/3}}\ln(\rho C\frac{V^2}{R}+1)$$

from volume=πD^2L/4, M=2E/V^2 = $\rho * volume$

This figure of merit is used because it does not depend on size. Essentially it is m of penetration per joule of KE. (Subscript s is for sand target.)

Penetrator Efficiency for varying velocity



There is a most efficient velocity, for this case of medium dense sand, about 60 m/s

$$\frac{P}{E^{1/3}} = \frac{(\rho f)^{2/3}}{\pi^{1/3} \rho_s C V^{2/3}} \ln(\rho C \frac{V^2}{R} + 1)$$

With constant velocity constraint, penetration scales as fineness ratio and density$^{2/3}$.

ARROW Penetration

Debate: is penetration a function of energy or momentum.
Equation shows it is a function of energy.
How you proportion that energy matters - put it in mass (density x length) or velocity. Almost certainly there is an optimum. But the best velocity for gelatin is not known. But over a range of useful velocities heavy and slow probably gives more penetration than light and fast. (Of course light and fast might be better for accuracy.)

$$\frac{P}{E^{1/3}} = \frac{(\rho f)^{2/3}}{\pi^{1/3}\rho_s C V^{2/3}} \ln(\rho C \frac{V^2}{R} + 1)$$

Momentum is proportional to ρf for constant D and V. But momentum cannot be isolated in the above equation, so penetrator efficiency is not dependent on momentum.