General Physics Framework for Penetration: Work, Energy, and Resistive Forces in Continuum Mechanics

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Date: November 2025

Abstract

This paper develops a comprehensive physics model of rigid-body penetration based solely on classical mechanics. Using the work–energy theorem, force–distance integration, and structural considerations, the model explains how kinetic energy converts into mechanical work against resisting forces in any continuum. The formulation is scale-independent, material-agnostic, geometry-neutral, and applicable across various contexts, offering a unified framework for predicting penetration behavior without relying on empirical or domain-specific assumptions.

Summary

When an object presses against a resisting material, it slows down because its kinetic energy is gradually consumed to overcome the forces resisting it. This paper explains this process in simple mechanical terms, showing how energy, force, mass, velocity, shape, and structural stability all affect how far an object can penetrate.

Velocity determines the total energy the object carries, while mass influences how quickly that energy is expended. The shape of the object affects how resistance increases as it moves deeper, and its ability to remain rigid ensures that energy isn't lost through bending or deformation.

The ideas presented here apply to any rigid or semi-rigid body entering any material. By focusing solely on general physical principles, the paper provides a clear and universal explanation of penetration without relying on specific tools, media, or real-world examples.

Table of Contents

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1. Introduction

Penetration often appears complicated because actual materials respond differently depending on their structure. However, the basic physics behind it is straightforward and universal. A penetrating object moves forward only by converting kinetic energy into mechanical work on the surrounding medium. The depth of penetration depends on how quickly resisting forces dissipate this energy.

This paper presents a comprehensive theoretical model applicable to any rigid body penetrating any resisting material. It does not depend on specific tools, geometries, or media. Instead, penetration is explained using concepts like energy, force, drag, structural integrity, and mechanical advantage. The aim is to provide a clear, self-contained physics explanation relevant across various fields—from engineering to biomechanics to impact mechanics.

This work was also motivated by discussions within the archery community that promoted a deeper understanding of the true mechanical principles behind penetration. Conversations with Joel Maxfiel, a well-known figure in modern archery, helped inspire the development of a comprehensive, first-principles approach to the topic.

Acknowledgment of Inspiration

This theoretical framework was partly inspired by discussions with Joel Maxfield, whose long-standing impact on modern projectile engineering and mechanical design encouraged a deeper investigation of the underlying physics. Although the analysis presented here is purely based on first principles and independent of specific equipment, his interest in understanding the true mechanical foundations of penetration helped inspire the development of this generalized model.

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2. Foundations of Penetration Physics

Penetration is controlled by a few universal physical principles. These principles explain how a moving object transfers energy, how that energy is lost through resisting forces, and how the object slows down as it goes deeper. The concepts in this section apply to all rigid bodies entering any resistive medium and form the basis for the generalized penetration model used in this paper.

2.1 Kinetic Energy: Total Work Capacity

A moving body with mass and velocity possesses kinetic energy:

$$KE = \frac{1}{2}mv^2$$

Kinetic energy represents the total amount of mechanical work an object can do before coming to a stop. Penetration ceases when this energy has been fully transformed into resistance work.

In simple terms, when the energy runs out, the motion comes to an end.

Key properties:

- KE increases linearly with mass.
- KE increases quadratically with velocity.
- The quadratic relationship with velocity makes speed the main factor in work potential.
- KE establishes the maximum allowable penetration depth through.

$$\int_0^D F(x)\,dx = KE$$

2.2 Momentum: Resistance to Deceleration

Momentum is defined as:

$$p = mv$$

Momentum indicates how much an object resists slowing down. Greater momentum means the object decelerates more slowly when opposing forces are applied.

Momentum influences:

- The shape of the deceleration curve.
- How is resistance force distributed over distance?

Momentum does **not** determine the maximum depth of penetration;
only energy can do that.

Momentum influences how an object comes to a stop. Energy decides where it ceases.

2.3 The Work–Energy Relationship: The Core of Penetration

Penetration is determined by the work–energy theorem:

$$\int_0^D F(x)\,dx = \frac{1}{2}mv^2$$

This means:

- Penetration continues as long as the available energy exceeds the resisting work.
- Penetration stops when resisting work equals the object's initial KE.
- This applies to all materials, shapes, and resistive environments.

This is the basic penetration equation.

2.4 Scalar and Vector Quantities in Penetration Mechanics

Misunderstanding scalars and vectors causes incorrect interpretations, especially the idea that kinetic energy "spreads out" because it lacks direction. Clarifying these concepts removes this misconception.

Kinetic Energy Is a Scalar

A scalar has only magnitude. It does not radiate, spread, or disperse in any direction. It simply lacks a directional label.

Kinetic energy tells you:

- How much work the object can perform.
- But not where that work will take place.

Direction is governed by forces and momentum, not by energy.

Momentum Is a Vector

Momentum has:

- Magnitude
- Direction

During penetration:

- Resisting forces act exactly opposite to the velocity vector.
- Momentum determines how fast velocity decreases.
- Momentum does not set the depth limit.

Why KE Does NOT Spread Out

Some readers assume:

Scalar = spread uniformly in all directions.

This is wrong.

A scalar has no directional component, so it cannot imply “all directions” or “sideways effects.”

Energy is used only in the direction of motion because:

- Resisting forces oppose the velocity vector.
- Work is defined as

$$W = \vec{F} \cdot \vec{d}$$

- Therefore, all available energy is directed forward along the penetration path.

Clear Contrast

Kinetic Energy (Scalar)

- Defines: maximum work and penetration limits.
- Does NOT determine direction or deceleration pattern.

Momentum (Vector)

- Determines: deceleration rate, velocity decline curve
- Does NOT define the maximum work available.

This resolves the common misunderstanding between scalars and vectors.

2.5 Components of the Resisting Force

The resisting force during penetration is represented as:

$$F(x, v) = F_s + F_c + F_d(v)$$

Where:

- F_s: shear-related resistance (creating or extending fracture surfaces)
- F_c: compressive resistance (displacing or deforming the medium)
- $F_d(v)$: velocity-dependent drag

Velocity-dependent drag typically follows:

$$F_d = \frac{1}{2}\rho C_D A v^2$$

These forces collectively govern the rate at which the projectile's kinetic energy is lost.

2.6 Deceleration Behavior

Deceleration results from Newton's second law:

$$m\frac{dv}{dt} = -F(x, v)$$

Using the chain rule:

$$mv\frac{dv}{dx} = -F(x, v)$$

Integration yields the velocity–distance curve, which universally:

- Drops quickly at first when drag is high.
- Declines more gradually at mid-depth.
- Tapers gradually as velocity approaches zero.

Total energy sets the stopping point; resisting forces shape the slowing process.

Section 2 Summary

- Kinetic energy determines the maximum possible penetration depth.
- Momentum determines how fast velocity drops.
- Scalars don't “spread”; they just lack directional labels.
- Energy is only used in the direction of motion since force determines the direction.
- Resisting forces determine how quickly energy is used.

3. The Penetration Equation

Penetration is a process of energy transfer: a moving object penetrates a resisting medium only by converting its kinetic energy into mechanical work. The opposing forces within the medium determine how quickly this energy is exhausted, and the work–energy theorem offers a complete mathematical description of the process. This section derives the general penetration equation, explains how resisting forces influence penetration depth, and highlights the fundamental energy limits.

3.1 Work Performed Against Resistance

As a rigid body enters a continuum, it encounters a resisting force that varies with penetration depth and, in some cases, with velocity. The work done against this force is:

$$W = \int_0^D F(x)\, dx$$

Where:

- D= total penetration depth
- $F(x)$= resisting force at depth x

This expression represents the total mechanical work required to stop the object. Since the body can perform only as much work as its initial kinetic energy, we equate:

$$\int_0^D F(x)\,dx = KE_{\text{initial}} = \frac{1}{2}mv^2$$

This is the **primary penetration equation**.

3.2 Energy–Resistance Balance and Penetration Depth

The penetration depth is not random; it is completely determined by the balance between:

1. **Initial kinetic energy** (work capacity), and
2. **The profile of resisting force (work done per unit distance).**

If an average value approximates the resisting force F_{avg}, the penetration depth simplifies to:

$$D = \frac{KE_{\text{initial}}}{F_{\text{avg}}}$$

This form highlights two central facts:

- Increasing kinetic energy increases penetration depth.
- Increasing the resisting force decreases penetration depth.

The work–energy principle does not assume any specific material type, projectile shape, or penetration media; it is a universally applicable law rooted in classical mechanics.

3.3 Components of the Resisting Force

To understand how the resisting force influences penetration, it can be broken down into basic components.

$$F(x, v) = F_s + F_c + F_d(v)$$

Where:

- F_s: shear-related resistance (creating or extending fracture surfaces)

- F_c: compressive resistance (displacing, deforming, or compacting material)
- $F_d(v)$: velocity-dependent drag

Drag typically follows:

$$F_d = \frac{1}{2}\rho C_D A v^2$$

This structure, shear + compression + drag, is universal.
Any resisting force can be expressed this way, whether the medium is soft, dense, laminated, granular, or heterogeneous. The energy cost of penetration is therefore determined by how these force components behave with depth.

3.4 Deceleration and Energy Dissipation

Deceleration happens because of the resisting force opposing the velocity vector.

$$m\frac{dv}{dt} = -F(x, v)$$

Using the chain rule:

$$mv\frac{dv}{dx} = -F(x, v)$$

Integrating this equation gives the velocity–distance curve:

$$\int_{v_0}^{0} mv\,dv = -\int_{0}^{D} F(x, v)\,dx$$

This shows that:

- Deceleration rate depends on momentum and resisting force.
- but the stopping distance depends on energy rather than momentum

This aligns with the scalar vector distinctions outlined in Section 2.

3.5 Core Insights from the Penetration Equation

Several key conclusions follow directly from the penetration equation:

1. Energy sets the maximum penetration depth

$$D_{\text{max}} = \frac{1}{F_{\text{avg}}}\left(\frac{1}{2}mv^2\right)$$

No amount of momentum can **overcome insufficient energy**.

2. Momentum controls deceleration, not depth

Momentum influences how slowly velocity decreases, but it does not alter the total amount of work the projectile can do.

3. Resistance determines how fast energy is consumed

Higher resisting forces decrease penetration depth by requiring more work per unit distance.

4. Velocity is the dominant contributor to penetration capability

Because velocity is squared in the energy term, increases in launch speed disproportionately increase work capacity.

5. The penetration equation is fully general

It applies to:

- any rigid or fin-stabilized body
- any resisting medium
- any geometry

- any speed regime
- any combination of shear, compression, and drag

This universality is a strength of the work–energy formulation.

Section 3 Summary

Penetration depth depends on the balance between initial kinetic energy and the resisting forces along the penetration path. The work–energy theorem offers the governing equation:

$$\int_0^D F(x)\,dx = \frac{1}{2}mv^2$$

Momentum influences how velocity shifts with depth, while kinetic energy determines the maximum possible penetration. Breaking down resisting forces into shear, compression, and drag helps analyze the penetration problem for any projectile and medium.

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4. Mass and Velocity in Penetration Mechanics

Mass and velocity are the two main properties of a moving object that affect how penetration happens. Although they are often discussed together, they influence penetration in fundamentally different ways. Velocity determines the total work potential through its quadratic effect on kinetic energy, while mass impacts the rate of deceleration by changing momentum. This section explains the distinct roles of each factor and clarifies how they shape the penetration process.

4.1 Mass: Linear Contribution to Energy and Inertia

Mass affects penetration in two important ways:

(1) Linear contribution to kinetic energy

From:

$$KE = \frac{1}{2}mv^2$$

Doubling the mass doubles the total kinetic energy if the **velocity remains constant**.
This directly increases the amount of work the object can perform against resisting forces.

(2) Increased resistance to deceleration

From Newton's second law:

$$m\frac{dv}{dt} = -F(x, v)$$

A larger mass causes smaller acceleration (or slower deceleration) when the resisting force stays the same. This shifts the velocity-distance curve, making the object keep its velocity longer into the medium.

Key point:

Mass affects **how the energy is used**; not how much energy is available (unless velocity is held constant).

4.2 Velocity: Quadratic Contribution to Energy

Velocity is the dominant factor in penetration because it contributes **quadratically** to kinetic energy:

$$KE \propto v^2$$

This means:

- +10% velocity → +21% energy
- +20% velocity → +44% energy
- +30% velocity → +69% energy

Small increases in velocity produce large increases in work potential.

Velocity also increases resistive drag

Drag typically follows:

$$F_d = \frac{1}{2}\rho C_D A v^2$$

So higher velocity increases both:

- the energy available for penetration
- the resisting force encountered early in penetration

However, because energy rises faster than drag, higher velocity generally increases penetration capability.

4.3 Mass vs. Velocity: Distinct Effects on Penetration

Mass and velocity do not play interchangeable roles. Their effects differ fundamentally:

Mass

- **Energy contribution:** Linear
- **Effect on deceleration:** Yes, greater mass slows deceleration (higher inertia and momentum)
- **Dominant effect:** Inertia and momentum
- **Meaning:** Mass changes *how fast* energy is used, not *how much* energy exists

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Velocity

- **Energy contribution:** Quadratic
- **Effect on deceleration:** Yes, higher velocity increases early-stage drag
- **Dominant effect:** Work potential
- **Meaning:** Velocity determines the total work available for penetration

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Critical Distinction

- **Velocity controls how much work is available.**
- **Mass controls how quickly the work is consumed.**

This distinction is crucial in understanding penetration depth.

4.4 Momentum vs. Kinetic Energy Revisited

As established earlier:

$$p = mv$$

Momentum determines how quickly the object slows down under a resisting force.
It influences the shape of the velocity-distance curve, not the maximum penetration depth.

Why momentum does not determine penetration depth

Penetration stops when:

$$KE_{\text{initial}} = \int_0^D F(x)\, dx$$

Once all kinetic energy is spent performing work against resistance:

- momentum also becomes zero
- penetration ceases
- momentum cannot extend penetration without energy

Momentum is important during penetration; energy determines how far penetration can go.

4.5 Equal-Energy Comparisons

If two bodies have equal kinetic energy:

- A heavier object must have a lower velocity.
- A lighter object must have a higher velocity.

Their **total work potential is identical**, but their deceleration behaviors differ:

- The heavier body slows more gradually (higher momentum).
- The lighter body slows more quickly (lower momentum).

However:

If resisting forces stay the same, objects with equal energy will penetrate to the same depth, because they each have the same total work capacity.

This does not depend on material type it follows directly from the work–energy theorem.

4.6 Key Insights

1. **Velocity dominates energy contribution** because of its quadratic effect.
2. **Mass increases energy only linearly** and primarily influences deceleration.
3. **Momentum shapes the deceleration curve** but **does not control penetration depth**.
4. **Penetration depth is set by energy**, not momentum.
5. **Mass affects how fast energy is used**, not how much energy exists.

Section 4 Summary

- Mass influences penetration by increasing energy (linearly) and reducing deceleration.
- Velocity influences penetration by increasing energy (quadratically) and increasing initial drag.
- Momentum affects the rate of slowing, not the final penetration depth.
- The work–energy theorem remains the governing principle behind how far penetration can continue.

This completes the analysis of how mass and velocity contribute to penetration mechanics.

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5. Leading-Edge Geometry and Penetration Efficiency

The front part of a penetrating object determines how it starts interacting with the resisting medium. Before any significant part can enter, the front shape must create enough localized stress to cause deformation or displacement in the material. The angle, curvature, and surface area of this front interface greatly affect the force needed to start and continue penetration.

This section explains the fundamental physical principles of stress concentration, displacement mechanics, and wedge mechanics that govern how any leading geometry interacts with a resisting continuum.

5.1 Stress Concentration at the Point of Entry

Penetration starts when the local stress applied by the leading edge exceeds the medium's capacity to resist deformation. Stress is represented by:

$$\sigma = \frac{F}{A}$$

Where:

- F= applied force
- A= effective contact area at the leading edge

Key Principle:

A smaller initial contact area results in higher local stress, decreasing the force required for the object to start entering the medium.

This applies to any leading-edge shape:

- pointed
- rounded
- flat but narrow
- tapered
- conical

The specific geometry does not matter; **smaller contact areas always raise local stress**, improving entry efficiency.

5.2 Displacement-Based Entry Mechanics

After initial entry, the leading geometry must keep displacing the surrounding material to enable deeper penetration. The necessary force depends on:

- how rapidly the geometry's diameter increases
- how much material needs to be displaced per unit distance
- how the medium responds to deformation

A gradual change in cross-sectional area decreases material displacement per unit length, reducing resistance. Conversely, a rapid expansion increases displacement rate, raising resistance.

Key Principle:

Geometries that gradually expand with distance require less force to sustain penetration.

This concept applies universally to all materials: liquid, soft solid, dense solid, granular, or composite.

5.3 Wedge Mechanics in Rigid and Semi-Rigid Media

If the medium provides substantial resistance, the leading geometry acts like a mechanical wedge. The object must exert a normal force to move material aside. For a wedge with angle θ:

$$F_{\text{required}} \propto \frac{1}{\tan(\theta)}$$

Interpretation:

- **Smaller (shallower) wedge angles decrease the forward force required to move forward.**
- **Larger (steeper) wedge angles need more force because they move more material with each unit of distance.**

Wedge mechanics apply to:

- rigid materials
- semi-rigid materials
- layered or laminated structures
- materials that do not readily deform without accumulating strain

This principle does not depend on any specific tool or object, only on the geometry and the medium's response.

5.4 How Leading Geometry Influences Resistive Force

The shape of the leading edge determines how the resisting force evolves with penetration depth. Specifically, it affects:

1. Initial force required to begin penetration

(smaller area → higher stress → easier entry)

2. Rate of material displacement as depth increases

(gentle taper → lower displacement rate → lower resistance)

3. Normal force acting between the object and the medium

(steep geometry → higher normal force → higher resistance)

4. How quickly energy is consumed

(more resistance → more work required per unit distance)

5. Stability of penetration path

(smoother geometries produce more predictable force curves)

Key Principle:

The advanced design dictates how quickly kinetic energy transforms into work, thereby influencing penetration depth.

5.5 Universal Principles of Leading-Edge Efficiency

Regardless of size, material, or application, the following principles hold for all penetrating bodies:

1. Higher stress at the leading edge reduces initial resistance.
2. Gradual expansion reduces the force required for continued entry.
3. Shallower wedge angles reduce normal and compressive resistance.
4. Efficient geometries expend energy more slowly.
5. Inefficient geometries increase resistance and shorten penetration.

These principles apply to:

- industrial penetrators
- ballistic probes
- impact tools

- mechanical plungers
- rigid bodies in fracture or displacement studies

The physics is independent of scale or application.

Section 5 Summary

- Entry begins when the leading edge generates enough localized stress to exceed the medium's resistance.
- The geometry of the leading interface affects how much material must be displaced as depth increases.
- Shallow or gradually expanding geometries reduce resistance, consuming less energy per unit distance.
- Steep or rapidly expanding geometries increase resistance and consume energy more quickly.
- Leading-edge shape plays a critical role in determining how efficiently kinetic energy is used during penetration.

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6. Body Interaction with the Penetration Channel

Once the leading edge begins to enter a resisting medium, the rest of the body faces a different set of mechanical forces. These forces act along the surface of the penetrating object and significantly influence how quickly the object's kinetic energy is used up. The shape, diameter, surface area, and surface roughness of the body determine the nature and strength of these forces.

This section explains the three universal force mechanisms that act on the main body during penetration: normal-force–driven friction, compressive displacement, and velocity-dependent drag.

6.1 Normal Force and Friction Along the Penetrating Body

As the body advances deeper into the medium, the surrounding material exerts a normal force $N(x)$ perpendicular to the surface. This normal force is a natural response of the medium resisting compression or displacement.

Friction follows:

$$F_f = \mu N(x)$$

Where:

- μ= coefficient of friction (depends on both surfaces)
- $N(x)$= normal force at depth x

Key Principles:

- Greater normal force → higher friction.
- Larger surface area → more contact → more friction.
- Rough surfaces increase μ, smooth surfaces decrease μ.

Friction rises with penetration depth because more of the body interacts with the surrounding medium.

6.2 Cross-Sectional Area and the Rate of Material Displacement

The resisting medium must be pushed outward to create space for the body. This displacement generates forces proportional to the cross-sectional area of the object.

For a cylindrical body:

$$A_{\text{contact}} = \pi d x$$

Where:

- d= body diameter
- x= penetration depth

Implications:

- Larger diameters cause more displacement, which increases resistance.
- Smaller diameters lead to less displacement, which reduces resistance.
- The relationship is linear, with penetration depth, and resistance generally increases as the object goes deeper.

This principle applies to all penetrating bodies regardless of shape or scale.

6.3 Velocity-Dependent Drag

Even after the front edge has entered, the moving object continues to face form drag from the medium. Drag is usually modeled as:

$$F_d = \frac{1}{2}\rho C_D A v^2$$

Where:

- ρ= density of the medium
- C_D= drag coefficient (depends on shape)
- A= cross-sectional profile presented to the flow or displacement
- v= velocity of the penetrating body

Key Behaviors:

- Drag is greatest at the beginning of penetration, when the velocity is at its highest.
- Drag lessens as speed drops, helping shape the universal deceleration curve.
- Drag acts opposite the direction of motion, directly dissipating kinetic energy.

Velocity-dependent drag is common across all penetration events, regardless of whether the medium acts like a fluid, soft solid, granular material, or something in between.

6.4 Cumulative Effect of Surface Forces

The total resisting force on the body after initial entry consists of three main components:

$$F_{\text{body}}(x, v) = F_f + F_c + F_d$$

Where:

- F_f= friction
- F_c= compressive displacement and normal resistance
- F_d= drag

Important Observations:

1. These forces grow with depth because more surface area is engaged.
2. Energy consumption accelerates as resisting forces grow.
3. The geometry of the body determines how quickly resistance increases.

Objects with streamlined or gradual sidewalls lose energy more slowly. Objects with abrupt transitions or large diameters lose energy faster.

6.5 Why Body Geometry Matters

The body behind the leading edge determines:

- how quickly friction increases
- how rapidly displacement forces grow
- how the drag coefficient behaves with depth
- whether resistance increases smoothly or abruptly
- how predictable the deceleration pattern is

General Principle:

The body does not determine whether penetration begins; it determines how efficiently penetration continues.

Well-designed bodies use energy gradually. Poorly efficient bodies burn energy quickly.

6.6 Section 6 Summary

- After initial entry, the body faces friction, resistance to displacement, and drag.
- These forces grow stronger with depth as more surface area interacts with the medium.
- Cross-sectional area and surface features significantly influence friction and work of displacement.
- Drag depends on velocity and shape, dominating early in penetration.
- Body geometry affects how quickly the object's kinetic energy is used up, directly impacting total penetration depth.

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7. Velocity–Distance Behavior During Penetration

As a penetrating object moves through a resisting medium, its velocity decreases in a predictable pattern determined by the balance between kinetic energy and the forces resisting motion. Although the specific magnitudes of these forces vary depending on the geometry and material behavior, the overall shape of the velocity–distance curve remains consistent across different penetration scenarios.

This section explains why velocity decreases the way it does, how resisting forces shape the deceleration profile, and why this pattern is fundamentally the same for all rigid-body penetration events.

7.1 Governing Equation of Deceleration

Deceleration follows directly from Newton's second law:

$$m\frac{dv}{dt} = -F(x, v)$$

Using the relationship $\frac{dv}{dt} = v\frac{dv}{dx}$, we obtain:

$$mv\frac{dv}{dx} = -F(x, v)$$

This equation precisely explains how velocity varies with penetration depth, considering the total resisting force.

The resisting force $F(x, v)$ consists of:

- velocity-dependent drag,
- normal-force-driven friction,
- compressive resistance,
- and any other medium-specific interactions.

Regardless of the medium, the deceleration rate depends on how these forces evolve as the object travels deeper.

7.2 Why the Velocity Curve Has a Universal Shape

All rigid-body penetration events show the same general pattern:

1. Rapid initial deceleration

At the moment of entry, velocity is at its peak, and drag, which increases with velocity, is at its highest. The object quickly loses a significant part of its kinetic energy during the initial phase of the penetration process.

2. Moderate mid-stage deceleration

As the object slows, drag lessens, and resistance is mainly caused by displacement and frictional forces. The velocity decreases more gradually.

3. Slow final approach to zero velocity

Near the stopping point, the object's velocity is low, so drag becomes negligible.
Velocity decreases slowly as friction and displacement forces consume the remaining kinetic energy.

Key Principle:

Velocity decreases quickly at first, then more gradually, and finally tapers as the body approaches rest.

This pattern is independent of the object's size, material, or geometry.

7.3 Influence of Velocity-Dependent Drag

Drag follows the general relationship:

$$F_d = \frac{1}{2}\rho C_D A v^2$$

This is why deceleration is steepest early in penetration:

- velocity is highest
- drag is proportional to v^2

- energy loss per unit distance is greatest

As velocity decreases:

- drag falls rapidly
- the resisting force shifts from drag-dominated to friction- and displacement-dominated

This transition creates the characteristic “curved” deceleration profile.

7.4 Influence of Friction and Displacement Forces

At lower velocities, friction and displacement forces dominate. These forces are not strongly dependent on velocity; instead, they depend on:

- surface area contacting the medium,
- normal force between the object and the medium,
- material response to displacement,
- rate of increase of cross-sectional area.

These forces grow with penetration depth but do not cause sharp velocity drops.
Instead, they create the gradual reduction in velocity seen in mid- and late-stage penetration.

7.5 Energy Perspective on the Velocity Curve

The velocity curve can also be understood directly from the work–energy theorem:

$$KE(x) = KE_0 - \int_0^x F(x')\,dx'$$

Where:

- KE_0= initial kinetic energy
- $F(x')$= total resisting force at position x'

Interpretation:

- Early in penetration, the resisting force is large → energy drops quickly → velocity drops sharply.

- As velocity decreases, energy is consumed more slowly → velocity curve flattens out.
- Penetration ends when energy reaches zero → velocity reaches zero.

This is a universal behavior governed entirely by classical mechanics.

7.6 Why Momentum Shapes the Curve but Not the Depth

As explained earlier:

- Momentum determines how sharply the object slows for a given resisting force.
- Kinetic energy determines how far penetration can continue.

Objects with higher momentum (at the same energy) decelerate more gradually, producing a flatter velocity curve but they still stop at the same depth once energy is exhausted.

This reinforces that momentum affects the shape of the curve, not the final penetration distance.

7.7 Section 7 Summary

- Deceleration is governed by $mv\frac{dv}{dx} = -F(x, v)$.
- All penetration events show a universal velocity–distance pattern: fast initial drop, moderate mid-range decline, slow taper to zero.
- Drag dominates early deceleration due to its v^2 dependence.
- Friction and displacement forces dominate late-stage deceleration.
- Kinetic energy determines the stopping distance; momentum determines the deceleration profile.

This completes the analysis of how velocity evolves during penetration.

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8. Structural Integrity and Energy Loss Mechanisms

A penetrating body performs work on the surrounding medium by transferring kinetic energy into mechanical resistance. This transfer is most efficient when the body maintains its structural integrity during penetration. Any deformation such as bending, buckling, folding, twisting, or fracturing diverts energy away from the penetration path and increases resisting forces.

This section explains how structural stability affects penetration mechanics and why preserving rigidity is essential for efficient energy use.

8.1 Structural Deformation as an Energy Sink

When a penetrating object deforms, some of its kinetic energy is absorbed internally instead of doing external work. The overall energy relationship becomes:

$$KE_{\text{initial}} = W_{\text{medium}} + W_{\text{deformation}} + W_{\text{lost}}$$

Where:

- W_{medium}= useful work done against resisting forces
- $W_{\text{deformation}}$= energy spent bending, buckling, or distorting the body
- W_{lost}= any additional losses (heat, friction at joints, vibrations, etc.)

Key Principle:

Deformation consumes energy that would otherwise contribute to penetration depth.

This is true for all rigid-body penetration events, regardless of the specific object or medium.

8.2 Increased Normal Force From Loss of Shape

When a body bends or buckles, its surface no longer aligns with the penetration path. This misalignment forces the object into greater contact with the surrounding material, producing larger normal forces.

Since friction is:

$$F_f = \mu N$$

an increase in normal force raises friction proportionally.

Consequences of loss of structural alignment:

- friction increases

- displacement resistance increases
- drag coefficient may increase
- total resisting force rises significantly

These effects accelerate energy loss and reduce penetration depth.

8.3 Surface Instability and Force Redirection

If the body deflects laterally during penetration, the resisting forces are no longer aligned opposite to the velocity vector. This creates additional components of force:

- side-loading
- off-axis bending
- torsional loading
- asymmetric compressive resistance

These components increase the total work required and consume energy rapidly.

In short:

Loss of alignment = loss of efficiency.

The object now has to do additional work just to keep its orientation or stay moving forward.

8.4 Buckling and Failure Thresholds

Slender or elongated bodies can buckle when subjected to compressive loads. The critical buckling force for a column is frequently estimated using Euler's formula:

$$F_{\text{critical}} = \frac{\pi^2 EI}{(KL)^2}$$

Where:

- E= modulus of elasticity
- I= second moment of area
- L= unsupported length
- K= effective length factor

Implications for penetration:

- If resisting forces exceed the critical load, the body will buckle.
- Buckling absorbs energy rapidly and drastically reduces penetration.
- Greater stiffness (higher EI) increases resistance to buckling.

Structural failure always redirects energy away from useful work.

8.5 Material Yielding and Plastic Deformation

If the body's material yields, the leading edge or main section may deform plastically. Plastic deformation requires a significant amount of energy:

$$W_{\text{plastic}} = \int_{\epsilon_y}^{\epsilon_f} \sigma \, d\epsilon$$

Where:

- ϵ_y= yield strain
- ϵ_f= final strain
- σ= stress during plastic flow

This energy is absorbed internally and does not contribute to overcoming external resistance.

Effects of plastic deformation:

- reduced structural stiffness
- increased surface area contacting the medium
- altered geometry leading to higher resisting forces

All of these reduce penetration efficiency.

8.6 Structural Rigidity as a Performance Multiplier

A structurally rigid body uses its kinetic energy more efficiently because:

1. It maintains consistent geometry.
2. It preserves alignment with the direction of motion.

3. It minimizes surface contact area growth.
4. It avoids internal energy losses from bending or yielding.
5. It prevents force redirection away from the penetration path.

General Rule:

The stiffer the penetrating object, the more of its kinetic energy can be used to perform useful external work.

This principle applies universally across engineering, biomechanics, materials science, and ballistic penetration research.

8.7 Section 8 Summary

- Structural deformation absorbs energy internally, reducing penetration capability.
- Loss of shape increases normal force and friction.
- Off-axis deformation causes side-loading that consumes additional work.
- Buckling and yielding can dramatically increase resistance or cause failure.
- Maintaining rigidity ensures that kinetic energy is used efficiently to overcome resisting forces.

This section completes the structural mechanics portion of the penetration framework.

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9. Mechanical Advantage and Load Distribution in Penetration

The efficiency of penetration depends not only on the energy carried by the object and the resisting force of the medium but also on how forces are distributed throughout the shape of the penetrating body. Mechanical advantage refers to a shape's ability to convert a smaller applied force into a larger effective force within the medium. In penetration mechanics, this idea involves wedge action, taper geometry, and the distribution of normal forces.

This section explains how mechanical advantage influences penetration efficiency, why specific geometries decrease the force needed, and how load distribution affects the work done during the medium's displacement.

9.1 Mechanical Advantage From Geometry

Mechanical advantage in penetration doesn't come from the object “riding up a ramp,” but from how geometry affects the distribution of normal forces needed to displace material. When a shape gradually changes diameter, the resisting medium is moved more smoothly, which lowers the instantaneous force required to keep moving.

Mechanical advantage comes from force distribution, not from lowering the needed force by sliding or ramping.

9.2 Distribution of Load on the Penetration Interface (Revised)

As the object moves forward, the resisting medium applies forces on the contact surface. If the surface expands slowly, the load increases gradually and uniformly. If it expands suddenly, the medium must generate higher resisting forces at specific points.

The main point is how smoothly the cross-sectional profile transitions, not a “ramp effect.”

Uneven load distribution not geometry itself is what increases resistance.

9.3 Force Vector Decomposition During Penetration

The resisting force can be decomposed into:

$$\vec{F}_{\parallel}\text{(opposing forward motion)}$$
$$\vec{F}_{\perp}\text{(normal to the surface)}$$

Geometry influences how much of the resisting force becomes normal versus axial. Efficient shapes minimize abrupt increases in the perpendicular component, keeping force aligned with the direction of travel.

This is **not** a reduction of force, but a **redistribution of how the medium reacts**.

9.4 Expansion Rate and Material Displacement (Revised)

The primary effect of geometry is how quickly the object forces the medium outward. A gradual expansion of diameter reduces the **rate of displacement**, which lowers resisting forces.

This is fundamentally different from a “ramp” model:

- We are discussing **rate of displacement**, not reducing required force via leverage.
- We are describing **material response**, not mechanical advantage from sliding.

Shapes that expand abruptly require more energy because they increase the displacement workload—not because the angle “lifts” anything.

9.5 Structural Support and Load Bearing

The ability of a geometry to maintain its shape under load influences its mechanical advantage. If portions of the body deflect or compress:

- the effective wedge angle changes
- the expansion rate becomes inconsistent
- load distribution becomes uneven
- resisting forces increase unpredictably

Key Rule:

Stable geometry preserves mechanical advantage; unstable geometry destroys it.

9.6 Universal Principles of Mechanical Advantage in Penetration

Mechanical advantage is not tied to any specific object. It applies universally whenever a shape interacts with a medium through displacement.

The governing principles are:

1. **Shallower angles generate larger normal forces for smaller input forces.**
2. **Rapid expansion increases displacement forces and energy consumption.**
3. **Uniform load distribution improves efficiency.**
4. **Force components depend on geometry; efficient shapes minimize perpendicular forces.**
5. **Structural rigidity is essential for maintaining mechanical advantage.**

These principles are valid in:

- impact penetration
- materials testing

- geological penetration
- industrial tooling
- mechanical engineering
- biomechanical studies

Physics is universal and independent of application.

9.7 Section 9 Summary

- Mechanical advantage arises from geometry converting small forward forces into larger displacement forces.
- Load distribution determines how efficiently the object interacts with the medium.
- Minimizing perpendicular force components reduces displacement resistance.
- Slow expansion and stable geometry preserve mechanical advantage.
- Efficient geometries consume less energy, increasing penetration depth.

Section 9 completes the last major variable influencing penetration efficiency.

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10. Conclusion

Penetration is a fully governed mechanical process that requires no assumptions beyond classical physics. A rigid body moves through a resisting medium only by converting its kinetic energy into mechanical work against opposing forces. These forces, drag, displacement, friction, and structural interaction, determine the rate of energy consumption. Penetration continues only as long as enough energy remains to overcome these forces.

Although mass and velocity both affect penetration, they do so in fundamentally different ways. Velocity determines the total work capacity through the quadratic dependence of kinetic energy, while mass affects the rate of deceleration via momentum. Confusing these roles leads to common misconceptions, such as the idea that kinetic energy "spreads out" because it is a scalar. In reality, energy is used strictly in the direction opposite the velocity vector where resisting forces act. At the same time, momentum shapes the velocity-distance curve but not the final depth of penetration.

The efficiency of penetration heavily depends on the shape and structural stability. The leading edge's shape determines how penetration begins by concentrating stress and affecting material displacement. Once an entry is established, the body behind the leading edge interacts with the surrounding environment through friction, regular forces, and velocity-dependent drag. A smooth, gradual change in geometry slows the growth of resisting forces, while a sudden expansion accelerates energy loss. Structural deformation, such as bending, buckling, or yielding, redirects energy internally and increases resistance, further limiting penetration depth.

Together, these principles create a complete and self-consistent framework: penetration depth depends on the total kinetic energy available, the development of resisting forces along the penetration path, and the ability of the penetrating body to retain its shape and alignment. This framework is universal. It applies to any rigid or semi-rigid body penetrating any resisting material, regardless of scale, application, or specific implementation.

By basing the analysis entirely on classical mechanics, including energy, force, deceleration, geometry, and structural behavior, this model offers a reliable, physics-based explanation of penetration that avoids unsupported assumptions, misconceptions, or non-physical interpretations. Anecdotal rules or application-specific theories do not drive penetration; it is governed by energy, resistance, and the mechanical interaction of shapes with materials.

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